ESTIMATE OF THE ULTIMATE DEFORMATION IN THE RUPTURE OF METAL PIPES SUBJECTED TO INTENSE LOADS

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A significant quantity of publications devoted to the investigation of the dynamics of expansion up to the rupture of metal pipes is known (see [1-5], for instance).

Considered below is the question of clarification of the physical regularity of the nonmonotonic nature of the change in the ultimate magnitude of the deformation of a cylindrical metal shell in the strain rate range 10^3-10^5 sec^{-1}. The results in [6-9] are closest to the idea of the energetic approach to the rupture problem developed here. In particular, commonality of the law for the existence of a "plasticity peak" for real metals during unsteady strain up to rupture is given a foundation.

1. Formulation of the Problem

A hollow metal cylinder subjected to internal pressure varying with time is expanded axisymmetrically until rupture under given initial data. There is no pressure on the shell outer boundary. Unsteady isentropic strain of the cylinder material is realized in a scheme of an isotropic continuous viscoplastic medium without vorticity.

The stress tensor components σ_r, σ_θ, σ_z, the radial component of the velocity vector v, and the density of the medium ρ in the ring under plane strain conditions are determined from a known system of equalities including the equation of motion of a continuous medium outside a mass force field, the continuity equation, and relationships of a viscoplastic medium [10, 11]. In formulating a mathematical model in an exact formulation, it is necessary to add to the above the equation of state and mass, momentum, and energy conservation laws before and behind the shock front (see, e.g., [12]). In a one-dimensional formulation of the experiment, the equation of state for metals is approximated satisfactorily by the equality

$$Q(\rho) = A[(\rho/\rho_0)^n - 1], \tag{1.1}$$

where A, n > 0 are constants determined in tests, and ρ_0 is the initial density of the material.

It is noted in [13, 14] that under impulsive loading of metallic shells the time of compression and unloading wave propagation and interaction over the shell thickness is less as compared with the total time of its deformation until rupture. Later, the wave nature of the change in density of the medium is not taken into account. We formulate averaging of the density with respect to time and thickness of the shell in the form of the following hypotheses:

a) The density of the material is homogeneous along the wall during unsteady axisymmetric expansion of a plane ring and is a function of just the time ρ(t), the specific case

$$\rho = \rho_0 (R_1/R_{10})^{\alpha_1} \tag{1.2}$$

is examined below;

b) The density distribution is independent of the time but changes along the thickness of the wall

$$\rho = \rho_0 (r/R_{10})^{\alpha_2}. \tag{1.3}$$

Here the dimensionless parameters α_1 and α_2 depend, according to (1.1)-(1.3), on the initial data of the loading process and are determined from the relationships

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$$\alpha_1 = -\frac{\dot{Q}(\rho_0)}{nA\dot{e}_{10}}, \quad \dot{e}_{10} = \dot{R}_1(0)/R_{10}, \tag{1.4}$$

$$\alpha_2 = \ln\left[\frac{Q(R_{20}) - Q(R_{10})}{A}\right]\Big/ n\ln(R_{20}/R_{10}),$$

where the dot denotes differentiation with respect to time t ≥ 0, R_{10}, R_{20} ($R_{10} < R_{20}$) are the inner and outer radii of the plane ring at the initial time, and $\dot{e}_{10} > 0$ is the initial strain rate of the medium on the ring inner surface.

Hypotheses a) and b) can have a real personification during the unsteady deformation of cylindrical shells and depend, in particular, on the method of fabricating these latter. In the general case, the real law of variation of the cylindrical shell material density over the shell thickness, on the average, still requires its determination during nonstationary motion.

Specifically, it is known that under uniaxial tension of metal specimens up to rupture under normal conditions, the material density diminishes, especially in the neighborhood of the rupture (neck) [15]. The diminution of the density under significant plastic deformations is explained by metal scientists by the formation of pores within and between the material grains (the so-called destruction) [16].

In turn, the metal density increases under the effect of intense shocks [12]. Therefore, because of the wave nature of compression and unloading wave interaction over the shell thickness in the initial stage of the motion, and then during intense deformation up to rupture, the law of material density variation can be approximated completely formally by the equalities (1.2)-(1.4). It is interesting to consider both cases in the subject of the influence of shell material compressibility during rupture. The possibility of comparing the results is conserved here in the scheme of an incompressible medium ($\alpha_1 = \alpha_2 = 0$).

2. Analytic Solution

Determination of the solution of the mathematical model for a plane ring with given boundary and initial conditions is simplified substantially under the hypotheses taken. In case a),

$$v = b_1 r^{-1} - \dot{e}_\rho r, \; \dot{e}_\rho = \dot{\rho}/2\rho, \; \rho = \rho(t), \tag{2.1}$$

$$b_1 = (\dot{e}_1 + \dot{e}_\rho) R_1^2, \; \dot{e}_1 = \dot{R}_1/R_1, \; R_2^2 - R_1^2 = (R_{20}^2 - R_{10}^2)\rho_0/\rho,$$

$$\dot{e}_1 + \dot{e}_\rho = (\dot{e}_2 + \dot{e}_\rho)\left(\frac{R_2}{R_1}\right)^2, \; \dot{e}_2 = \dot{R}_2/R_2,$$

$$\sigma_r = (\sigma_s + \rho\dot{b}_1)\ln r/R_2 + 0{,}5\rho\left[b_1^2(r^{-2} - R_2^{-2}) + (\ddot{e}_\rho + \dot{e}_\rho^2)(r^2 - R_2^2)\right] - 2\mu b_1(R_2^{-2} - r^{-2}),$$

$$\sigma_\theta = \sigma_r + \sigma_s + 4\mu b_1 r^{-2}, \; \sigma_z = \sigma_r + 0{,}5\sigma_s + 2\mu b_1 r^{-2},$$

$$p_0\left(\frac{R_{10}}{R_1}\right)^{2\gamma} = (\sigma_s + \rho\dot{b}_1)\ln R_2/R_1 + \rho\,(\ddot{e}_\rho + \dot{e}_\rho^2)(R_2^2 - R_1^2)/2 + b_1(\rho b_1/2 + 2\mu)(R_2^{-2} - R_1^{-2});$$

in case b),

$$v = b_2 r^{-(\alpha_2+1)}, \quad \rho = \rho_0 (r/R_{10})^{\alpha_2}, \tag{2.2}$$

$$b_2 = \dot{R}_1 R_1^{\alpha_2+1}, \quad \dot{e}_1 = \dot{e}_2 (R_2/R_1)^{\alpha_2+2}, \; R_2^{\alpha_2+2} - R_1^{\alpha_2+2} = R_{20}^{\alpha_2+2} - R_{10}^{\alpha_2+2},$$

$$\sigma_r = (\sigma_s + \rho_0 R_{10}^{-\alpha_2}\dot{b}_2)\ln r/R_2 + b_2\left(\frac{\alpha_2+1}{\alpha_2+2} b_2\rho_0 R_{10}^{-\alpha_2} + 2\mu\right)\left[r^{-(\alpha_2+2)} - R_2^{-(\alpha_2+2)}\right],$$

$$\sigma_\theta = \sigma_r + \sigma_s + 2\mu\,(\alpha_2 + 2)\, b_2 r^{-(\alpha_2+2)},$$

$$\sigma_z = \sigma_r + 0{,}5\sigma_s + \mu(\alpha_2 + 2)\, b_2 r^{-(\alpha_2+2)},$$

$$p_0\left(\frac{R_{10}}{R_1}\right)^{2\gamma} = \left(\sigma_s + \rho_0 R_{10}^{-\alpha_2}\dot{b}_2\right)\ln R_2/R_1 + b_2\left(\frac{\alpha_2+1}{\alpha_2+2} R_{10}^{-\alpha_2} b_2\rho_0 + 2\mu\right)\left[R_2^{-(\alpha_2+2)} - R_1^{-(\alpha_2+2)}\right].$$

Here, σ_S is the dynamic yield point; μ, coefficient of dynamic viscosity; r, θ, z, cylindrical coordinate system and, p_0, initial pressure on the ring inner boundary which diminishes during expansion of the boundaries of a shell with the isentropic index $\gamma > 1$.

Systems (2.1) and (2.2) agree for an incompressible medium ($\alpha_1 = \alpha_2 = 0$). For $\mu = 0$, $\alpha_1 = \alpha_2 = 0$ we obtain the known solution for axisymmetric expansion of a cylindrical shell in an ideal plasticity scheme [17] from (2.1), (2.2). When $\mu = 0$, $\sigma_S = 0$, $\alpha_1 = \alpha_2 = 0$, we have the case of an ideal incompressible fluid [18] from (2.1), (2.2). Taking account of the relationships between the running values of the ring radii R_1 and R_2 and the initial data, the last equations in (2.1), (2.2) determine the law of ring boundary variation in time. If we use the notation $y(R_1) = \dot{R}_1 R_1$, then we obtain a nonlinear first-order differential equation in $y(R_1)$ (an Abel equation of the second kind [19]) from (2.1) and (2.2). For a thin-walled ring this equation has a small parameter in the derivative. Taking the results of [20] into account, we find the asymptotic of the solution in the form of a series in the small parameter to a given degree of accuracy.

A numerical computation is represented in [14, 21] for $\alpha_1 \neq 0$, $\alpha_2 \neq 0$, $\mu = 0$, and comparisons are made with known data for a ring of elastic–plastic compressible material [22], where the wave pattern in the shell is taken into account.

3. Rupture Criteria

The main purpose of this paper is to clarify the singularities in the deformation of metal pipes up to rupture under the effect of significant dynamic loads. Rupture is a complex problem. We use here the known criterion of the fracture mechanics of solids in the form of the integral relationship [6]

$$\int q(V)\,dV = \lambda_* s_* \tag{3.1}$$

where V is the volume of the part of the body being rupture from which the elastic energy needed for rupture is taken; q is the density of the elastic energy liberated during unloading of the elastic wave; and λ_* is the work expended during the rupture of a unit section s_* of a solid.

Let us make the trajectory of rupture front propagation specific. Namely, tests show that metal pipes rupture during unsteady axisymmetric expansion when radial cracks or a system of radial cracks propagates along the wall. The development of each crack is accompanied by unloading along a circle being propagated at the speed of sound c [3]. The unloading wave progresses a distance $d\ell = c\,dt$ on both sides of the rupture section in the time dt. The tension energy being liberated is expended in development of the crack [6]. During this time the ring inner boundary is expanded by the quantity $dR_1 = \dot{R}_1 dt$, such that $d\ell = c\,dR_1/\dot{R}_1$.

Propagation of a radial crack over the ring thickness and simultaneous expansion of its boundaries permit determination of the rupture along the radius by the expression

$$dr = (v_c - \dot{R}_1)dt, \tag{3.2}$$

where v_c is the rate of crack propagation over the ring which depends on the crack length and other parameters of the rupture process (see, e.g., [23]). Hence, in the plane case we have $dV = 2d\ell dr$ for the desired volume in (3.1).

The question of a quantitative estimate of v_c for a viscoplastic material in a deformable ring remains open. The problem is made complicated by the fact that propagation of a radial separation crack in a cylinder loaded by internal pressure occurs in a material in a variable stress field and up to the time of the crack approach the medium under consideration undergoes significant strain. It is here necessary to formulate assumptions that do not contradict the real process. For instance, making the boundary conditions specific at the crack apex would permit a numerical investigation of the propagation process for a system of radial separation cracks in a cylinder wall [24].

In our case we assume that the process of loss of continuity of the material during axisymmetric expansion of the ring boundaries starts from the inner boundary and the rupture front is later propagated over the whole radial section of the ring (see, e.g., [3]). We use the notation $R_{1*} = R_1(t_*)$ for the value of the ring inner radius at the time of material rupture $t_* > 0$. For $t \in [0, t_*]$ $v_c = 0$, while for $t > t_*$ we have $v_c > 0$ and, as a rule, through rupture of the shell. From the physical viewpoint, this latter means that up to a definite value of the deformation time t_* (or the relative strain $e_{1*} = R_{1*}/R_{10} - 1$), crack growth to the critical value occurs at a microlevel in the material, then (for $t > t_*$) the crack is propagated intensely over the thickness of a shell known to be taking the latter out of the efficient class.

We consider below the estimate of the parameter t_* that characterizes the initial rupture process and does not require details of the law about the crack propagation velocity over the whole section of the ring. The passage to a one-dimensional scheme to determine dr from (3.2) permits writing $dr = R_1 - R_{10}$; then the domain dV enclosed by unloading is determined by the dependence

$$dV = 2(R_1 - R_{10})cdt, \tag{3.3}$$

where the velocity of the circumferential unloading waves in the ring is taken equal to the isentropic speed of sound in the material under consideration.

By definition, $c^2 = \partial Q/\partial\rho$, and we have from (1.1)-(1.3) for the hypotheses a) and b), respectively,

$$c = c_0\left(\frac{R_1}{R_{10}}\right)^{a_1}, \quad c_0 = \sqrt{\frac{An}{\rho_0}}; \tag{3.4}$$

$$c = c_0\left(\frac{r}{R_{10}}\right)^{a_2}, \quad a_i = \alpha_i(n-1)/2, \quad i = 1, 2.$$

It is known [25] that the specific elastic energy of a solid is found in the plane case from the expression

$$q = T_n^2(1-\nu^2)/2E. \tag{3.5}$$

Here ν is the Poisson ratio; E, Young's modulus; T_n, invariant of the tangential stress intensity which is determined for a viscoplastic body by the dependence [10]

$$T_n = \sigma_s/2 + \mu H,$$

where H is the invariant of the strain rate intensity and from (1.2) and (2.1),

$$H = (2 - \alpha_1)\dot{e}_1(R_1/r)^2; \tag{3.6}$$

while from (1.3) and (2.2),

$$H = (2 + \alpha_2)\dot{e}_1(R_1/r)^{\alpha_2+2}. \tag{3.7}$$

Taking account of the equality $s_* = R_{1*} - R_{10}$, for r = R the formulas (3.3)-(3.7) permit representation of (3.1) in the form of the integral equation

$$\int_0^{t_*}\left[\frac{\sigma_s}{2} + \mu\left(2 + (-1)^i\alpha_i\right)\dot{e}_1\right]^2\left(\frac{R_1}{R_{10}} - 1\right)\left(\frac{R_1}{R_{10}}\right)^{a_i}dt = \frac{\lambda_* E e_{1*}}{c_0(1-\nu^2)}, \tag{3.8}$$

where i = 1 corresponds to case a) and i = 2 to b). The relationship (3.8) determines the rupture time t_*. The dependence $R_1(t)$ is found here from the last equations of the system (2.1), (2.2).

An integral of the type (3.1) was examined in [6] under the assumption of constancy of the rate of expansion of the outer ring boundary ($\dot{R}_2$ = const). We consider two actual cases below: $\dot{e}_1$ = const and $\dot{R}_1$ = const. Here, the velocity $\dot{R}_1$ increases in the initial stage of cylindrical shell boundary expansion under the action of explosion products, and the ring radius grows correspondingly, which permits making the approximation $\dot{e}_1 = \dot{R}_1/R_1 \approx$ const. Taking into account the relationship $dt = dR_1/\dot{e}_1R_1$, we obtain the following equation for $e_{1*} = R_{1*}/R_{10} - 1$ from (3.8) for $\dot{e}_1$ = const, $a_i \neq 0$, $a_i \neq -1$

$$\frac{(1+e_{1*})^{a_i}}{a_i}\left[\frac{a_i(1+e_{1*})}{a_i+1} - 1\right] - \frac{1}{a_i}\left[\frac{a_i}{a_i+1} - 1\right] = \frac{Be_{1*}\dot{e}_1}{\left|\frac{\sigma_s}{2} + \mu\left(2 + (-1)^i\alpha_i\right)\dot{e}_1\right|^2} \tag{3.9}$$

$$(B = E\lambda_*/[c_0(1-\nu^2)]). \tag{3.9}$$

Taking account of the approximation to second-order accuracy in $e_{1*} < 1$,

$$(1+e_{1*})^{a_i} \simeq 1 + a_i e_{1*} + a_i(a_i - 1)e_{1*}^2/2,$$

we find from (3.9)

$$e_{1*} \simeq \frac{2B\dot{e}_1}{\left[\frac{\sigma_s}{2} + \mu\left(2 + (-1)^i \alpha_i\right)\dot{e}_1\right]^2}. \tag{3.10}$$

Analysis of (3.10) shows that there exists a nonmonotonic dependence of e_{1*} on the strain rate $\dot{e}_1$. The deduction of the existence of a plasticity peak is conserved in both cases i = 1, 2. From (1.4) we have for i = 1 a dependence of α_1 on the strain rate, for i = 2 there is no such dependence. Thus, from (1.4) and (3.10) for i = 1

$$e_{1*,\max} = \frac{(2+\alpha_1)B}{4\mu\sigma_s} \tag{3.11}$$

for

$$\dot{e}_1 = \frac{\sigma_s}{2(2+\alpha_1)\mu},\ \alpha_1 = 2\left(\frac{nA\sigma_s}{4\mu\dot{Q}(\rho_0^{\cdot})} - 1\right).$$

For i = 2 we obtain from (1.4) and (3.10)

$$e_{1*,\max} = \frac{B}{(2+\alpha_2)\mu\sigma_s} \quad \text{for } \dot{e}_1 = \frac{\sigma_s}{2(2+\alpha_2)\mu}. \tag{3.12}$$

When the ring inner boundary expands at a constant rate ($\dot{R}_1$ = const), the integration of (3.8) with respect to the variable $dt = dR_1/\dot{R}_1$ results in another form of the equation as compared with (3.9). However, by conserving the degree of approximation (to the second order) in the expansion of binomials in e_{1*}, we obtain (i = 1, 2) the expression (3.10) exactly, where $\dot{e}_{10} = \dot{R}_1/R_{10}$ must be taken in place of $\dot{e}_1 = \dot{R}_1/R_1$.

Therefore, the commonality has been confirmed of the existence of a plasticity peak in the case of deformation for not only a cylinder with a constant rate in the scheme of a viscoplastic incompressible medium [6], but also for a cylinder of compressible viscoplastic material in the regimes of inner boundary motion $\dot{e}_1$ = const and $\dot{R}_1$ = const.

4. Discussion

In order to compare the values of the ultimate strain calculated by means of (3.10)-(3.12) with experiment, the coefficient of dynamic viscosity and the yield point of the material under consideration must be known, and the density distribution law in the shell for significant compressibility of the material. It is known [26, 27] that μ, σ_s depend on the strain rate; especially significant is the coefficient of viscosity [28]. Quite limited information for μ is obtained in the 10^3-10^6 $\sec^{-1}$ strain rate range (see, e.g., [10, 29-32]). Hence, it appears to be of practical and scientific interest to estimate the dynamic viscosity coefficient μ of metals from the e_{2*}, $\dot{e}_2$, σ_s, λ_* known from test data.

Since mainly quantities corresponding to the ring outer boundary must be worked within tests on the dynamic rupture of pipes, we write the relationships useful later for a thin-walled cylindrical shell from (2.1) and (2.2) for $\dot{\rho} = 0$

$$\begin{gathered} e_{1*} \simeq e_{2*}(1+2\varepsilon_0),\ e_{2*} = R_{2*}/R_{20} - 1,\ \varepsilon_0 = \delta_0 \mid R_{10}, \\ \dot{e}_1 \simeq \dot{e}_2\left[1 + 2\varepsilon_0(1+e_{2*})^{-2}\right], \end{gathered} \tag{4.1}$$

where δ_0 is the initial width of the ring wall.

Then because of (4.1) for an incompressible viscoplastic medium ($\alpha_1 = 0$, $\alpha_2 = 0$, $\nu = 1/2$) we obtain from (3.10) to the accuracy of the first order of smallness in ε_0 the expression

$$\mu = \left[\frac{2\lambda_* E}{3c_0 e_{2*}\dot{e}_2}\right]^{1/2} \left\{1 - \frac{D}{2} + \varepsilon_0\left[\frac{(D-1)}{(1+e_{2*})^2} - 1\right]\right\}, \tag{4.2}$$

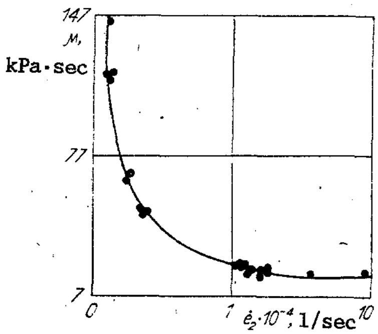

Fig. 1

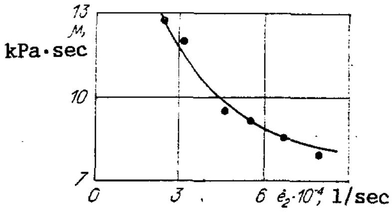

Fig. 2

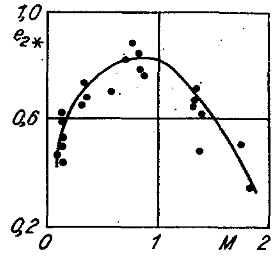

Fig. 3

$$D = \frac{\sigma_s}{2}\left[\frac{3e_{2*}c_0}{2E\lambda_* \dot{e}_2}\right]^{1/2}. \qquad (4.2)$$

Let us compute μ by means of (4.2) on the basis of test data on the explosive rupture of steel pipes [6, 33]. Represented in Figs. 1 and 2 are values of the dynamic viscosity coefficient for low-carbon steel (Fig. 1 is from the data in [6], $\sigma_S \approx 0.25$ GPa) and for Z3CN18-10 steel (Fig. 2, from data in [33], $\sigma_S \approx 0.33$ GPa) as a function of the strain rate. Taken in the computations for the steels under consideration were E = 200 GPa, $c_0 = 4.6$ km/sec, $\lambda_* = 210$ kJ/m². The diminution of μ due to the strain rate follows from Figs. 1 and 2, which is in agreement with the general deductions [26, 28] relative to the viscosity of metals. A quantitative comparison of the data in Figs. 1 and 2 with known estimates for steel [10, 28-31] relative to μ shows the nearby values for the corresponding strain rates.

Let us examine the structure of the dimensionless parameter $w = \mu\dot{e}_1/\sigma_S$ in more detail. Substituting it into (3.10) shows that the ultimate strain of the ring material also has an extremum in this parameter. Since the viscosity coefficient and the yield point of real metals depend on the change in the strain rate [26-28] the relationships

$$\sigma_s = \sigma_{s0}\left(1 + a_0 x^{n_1}\right), \; \mu = \mu_0 x^{-n_2}, \; x = \dot{e}_1/\dot{e}_{10} \geqslant 1 \qquad (4.3)$$

can provisionally be taken for the dynamic characteristics, where n_1, n_2, a_0 are positive constants; σ_{S0} and μ_0 are mechanical characteristics of the metal that are obtained during tests for the accessible strain rate $\dot{e}_{10} \le \dot{e}_1$. Formulas (4.3) reflect experimental observations: insignificant growth of the yield point and even a reduction in a number of cases when the strain rate increased (an almost linear dependence) and hyperbolic nature of the diminution in the viscosity coefficient of metals (see, e.g., [28] and Figs. 1 and 2). Then we obtain the following equality for the parameter w from (4.3):

$$w = \left(\frac{\mu_0 \dot{e}_{10}}{\sigma_{s0}}\right)\left[\frac{x^{1-n_2}}{1 + a_0 x^{n_1}}\right]. \qquad (4.4)$$

Analysis of (4.4) for the variation of x shows that the dependence w(x) has extremal values for definite parameters n_1, n_2, a_0. For instance, for $n_1 = 1$ and $n_2 = a_0 = 1/2$,

$$w_{\max} = \left(\frac{\mu_0 \dot{e}_{10}}{\sigma_{s0}}\right)\Big/\sqrt{2} \quad (x = 2).$$

Therefore, the existence of the maximal value for metal strain at the time of shell rupture is a characteristic of the material as a function of the strain rate. This is explained by the different intensity in the change in strength properties of a metal (σ_S) and internal friction (μ) during unsteady strain, which indeed governs the plasticity peak.

If test results relative to the dependence $e_{2*}(M)$ are considered [34, 35] (M is the ratio between the weight of the explosive substance and the weight of the cylindrical shell per unit length), we obtain curves with a maximum. For instance, according to experiments on the high-velocity rupture of thin-walled steel pipes [34] we find the location of the test points along the curve that have the maximum $e_{2*} \approx 0.82$ for $M \approx 0.87$ (Fig. 3). We obtain analogous curves with an extremum after treating the test data on explosive rupture of duraluminum, lead, copper, and brass pipes [35].

The fact noted here about the existence of extremal values for the functions $e_{2*}(\dot{e}_2)$, $e_{2*}(M)$ can apparently be explained by the presence of a single-valued monotonic functional dependence between the parameters $\dot{e}_2$, M, as the energetic estimate of the velocity of cylindrical shell motion under explosive loading [1] indicates.

Therefore, the analytic dependences found on the basis of the energetic rupture criterion (3.1) do not contradict the known test data on the rupture of metal pipes subjected to the products of an explosion, and also permit an actual prediction of the value of the plasticity peak during unsteady strain of metallic cylindrical shells in a more general formulation. In particular, it follows from (1.2)-(1.4), (3.11), (3.12) that different density distribution laws for the ring material affect the quantity $e_{1*,max}$ differently. For example, for the validity of (1.2) the parameter $\alpha_1 > 0$ increases the value of $e_{1*,max}$ while for (1.3) the $e_{1*,max}$ diminishes as the parameter α_2 grows. In both cases, taking account of the material compressibility reduces the value of the strain rate for which the plasticity peak is achieved. Such estimates are needed in practice to select the technological mode for fabrication in the dynamics of metal pipes or in the production of new structural forms of cylindrical pressure vessels.

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